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SEISMIC RISK ASSESSMENT OF A REINFORCED CONCRETE FRAME

EXAMPLE APPLICATION OF THE SAC/FEMA 2000 METHOD

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Abstract

An extension of the SAC/FEMA 2000 Seismic Risk Assessment Method has been applied to a reinforced concrete frame. The results of the example application confirm the applicability of the method to the case of reinforced concrete structures having multiple failure modes, as opposed to the original implementation of the method where a single failure mode is considered.

1. Introduction

Seismic assessment is a topic that captures civil engineers' on-growing attention worldwide, mainly because of the number of existing structures constructed according to 'old' inadequate building criteria, less severe for what concerns design details. Reliability-based methods for evaluating the risk are now the object of widespread research, where the random character of the parameters involved in the procedure (variability of the seismic event, randomness on the structural demand and capacity etc) is explicitly taken into account, as opposed to deterministic-based methods. So far, in order to foresee reliability approaches implemented in structural codes, both the computational effort and the degree of complexity of the models presented in the literature, must be alleviated.[1]

The main advantages of the SAC/FEMA 2000 Seismic Risk Assessment Method – an extension of which is discussed in the present report – are exactly the simplicity of the model and the gain on computational effort (drastic reduction of the number of non-linear dynamic analyses performed), considering realistic hypotheses and simplifications; the model becomes very attractive for engineers not specialized in the field, who are willing to obtain realistic results for practical use.

The method has been presented in [1], [2] and [3] and it is a development over a previous proposal (see [4], [5] and [6]). It has been conceived primarily in order to assess the seismic performance of steel structures. As it will be shown through the present report, an extension to reinforced concrete structures with multiple mechanisms of failure is feasible.

2. SAC/FEMA 2000 Method: Extension

The method, as presented in [1], [2] and [3], deals with a structure where a single failure mechanism is considered. While such a simplification can be acceptable for a steel structure, it is not realistic for a reinforced concrete one.

Thus, a generalization of the method consists in:

1. choosing multiple failure mechanisms, having a realistic possibility of developing in the structure;
2. choosing a Damage Measure (DM) parameter for each failure mechanism;
3. computing the individual failure probabilities for each critical element of the structure, accounting for every possible failure mechanism occurring in them;
4. evaluating the total seismic risk of the structure.

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[1] A review on several recent studies is presented in [12].

The seismic risk $p_{i,j}$ is defined as the annual probability[2] that the i-th critical element fails, due to the j-th mechanism of failure:

$$p_{i,j} = P(D_{i,j} \geq C_{i,j})$$

where $D_{i,j}$ is the demand and $C_{i,j}$ the capacity in the i-th element, related to the DM parameter associated to the j-th mechanism of failure.

The demand can be expressed as a function of an intensity measure (IM) parameter of the seismic event:

$$D_{i,j} = f_{i,j}(IM)$$

The IM parameter itself is related to the seismic hazard at the site under consideration. The seismic hazard can be defined as the annual probability that the intensity IM exceeds a value i_m:

$$H_{IM}(i_m) = P(IM \geq i_m)$$

In order to develop mathematically treatable expressions for the three aspects seen above, four assumptions are necessary:

- The Hazard Function is a random variable, log-normally distributed, with median $\hat{H}_{IM}(i_m)$ and dispersion $\beta_{H,U}$ due to uncertainty:

  $$H_{IM}(i_m) = \hat{H}_{IM}(i_m) \cdot \varepsilon_{H,U},$$

  where $\varepsilon_{H,U}$ is a log-normally distributed random variable, with unit median and dispersion $\beta_{H,U}$;

- The median hazard is expressed in the following simplified form:

  $$\hat{H}_{IM}(i_m) = k_0 \cdot i_m^{-k},$$

  where the coefficients k_0 and k are to be evaluated by regression, for the specific site where the structure is located;

- Demand $D_{i,j}$ and Capacity $C_{i,j}$ (relative to the DM level in the j-th mechanism of the i-th element) are independent log-normally distributed random variables, with medians $\hat{D}_{i,j}$ and $\hat{C}_{i,j}$, and dispersions, due to both randomness and uncertainty[3]:

  $$D_{i,j} = \hat{D}_{i,j} \cdot \varepsilon_{D_{i,j},R} \cdot \varepsilon_{D_{i,j},U}$$
  $$C_{i,j} = \hat{C}_{i,j} \cdot \varepsilon_{C_{i,j},R} \cdot \varepsilon_{C_{i,j},U},$$

  where $\varepsilon_{D_{i,j},R}$, $\varepsilon_{D_{i,j},U}$, $\varepsilon_{C_{i,j},R}$ and $\varepsilon_{C_{i,j},U}$ are log-normally distributed random variables, with unit medians and dispersions $\beta_{D_{i,j},R}$, $\beta_{D_{i,j},U}$, $\beta_{C_{i,j},R}$ and $\beta_{C_{i,j},U}$ respectively (R stands for randomness and U for uncertainty).[4]

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[2] Strictly, the mean annual frequency

[3] Randomness (or aleatory uncertainty) is the variability 'inherent' in a specific set of data, while uncertainty (or epistemic uncertainty) is the variability due to information available for the specific set.

[4] In extension, if different sources of randomness/uncertainty are considered in demand/capacity, the variable $\varepsilon_{D/C,R/U}$ can be expressed as the product of the (log-normally assumed distributed variables), $\varepsilon_{D/C,R_1/U_1}$, $\varepsilon_{D/C,R_2/U_2}$, … , where indices 1, 2, …, stand for 'source N°1, source N°2' etc.

- The median demand can be expressed as a function of i_m as:

$$\hat{D}_{i,j} = a_{i,j} \cdot i_m^{\ b_{i,j}}$$

where the coefficients $a_{i,j}$ and $b_{i,j}$ are to be evaluated by means of a certain number of non-linear dynamic analyses.

Based on these three assumptions, through a number of passages (see appendix I), the failure probability $p_{i,j}$ (of the j-th mechanism in the i-th element) is found to be a random variable itself, log-normally distributed, with median $\hat{p}_{i,j}$ and dispersion $\beta_{p_{i,j},U}$, due to uncertainty:

$$p_{i,j} = \hat{p}_{i,j} \cdot \varepsilon_{p_{i,j},U},$$

where $\varepsilon_{p_{i,j},U}$ is a random variable, log-normally distributed with unit median, as it is the product of log-normally distributed random variables with unit medians:

$$\varepsilon_{p_{i,j},U} = \varepsilon^{\frac{k}{b}}{}_{D_{i,j},U} \cdot \varepsilon^{\frac{-k}{b}}{}_{C_{i,j},U} \cdot \varepsilon_{H,U}.$$

The median value $\hat{p}_{i,j}$ is equal to the median hazard corresponding to the spectral acceleration producing the median capacity at DM, times an amplification factor depending on the dispersions due to randomness of both demand and capacity:[5]

$$\hat{p}_{i,j} = \hat{H}_{IM}\left(i_m(\hat{C}_{i,j})\right) \cdot \exp\left(\frac{k^2}{2 \cdot b^2}\left(\beta^2_{D_{i,j},R|i_m(\hat{C}_{i,j})} + \beta^2_{C_{i,j},R}\right)\right).$$

As for the dispersion $\beta_{p_{i,j},U}$, it is equal to the dispersion of $\varepsilon_{p_{i,j},U}$:

$$\beta_{p_{i,j},U} = \sqrt{\frac{k^2}{b^2} \cdot \beta^2{}_{D_{i,j},U} + \frac{k^2}{b^2} \cdot \beta^2{}_{C_{i,j},U} + \beta^2{}_{H,U}}.$$

After repeating the same procedure for all structure's critical elements and failure mechanisms considered, the total annual probability of failure can be evaluated, by considering the (mutually exclusive and collectively exhaustive) failure events as a series system:[6]

$$P_f = 1 - \prod_{i=1}^{N_e} \prod_{j=1}^{N_m} \left(1 - p_{i,j}\right),$$

where N_e and N_m are the number of critical elements and failure mechanisms respectively considered.

The total seismic risk P_f is also a random variable log-normally distributed, with median $\hat{P}_f$ and dispersion (due to uncertainty) $\beta_{P_f,U}$ (1[st] order approximation) given by:

$$\hat{P}_f = 1 - \prod_{i=1}^{N_e} \prod_{i=1}^{N_m} \left(1 - \hat{p}_{i,j}\right),$$

$$\beta_{P_f,U} \cong \hat{P}_f \cdot \left(\sqrt{\sum_{i=1}^{N_e} \sum_{j=1}^{N_m} \frac{\hat{p}^2{}_{i,j}}{\beta^2{}_{p_{i,j},U}}}\right)^{-1}.$$

[5] The demand dispersion value is calculated at the vicinity of the IM parameter value producing the median capacity.

[6] This assumption becomes less realistic for a structure with a high degree of hyperstaticity.

3. Example Application

The extension of the SAC/FEMA 2000 Method, presented in the previous paragraph, has been applied in the evaluation of the seismic risk of the reinforced concrete frame shown in fig.1.

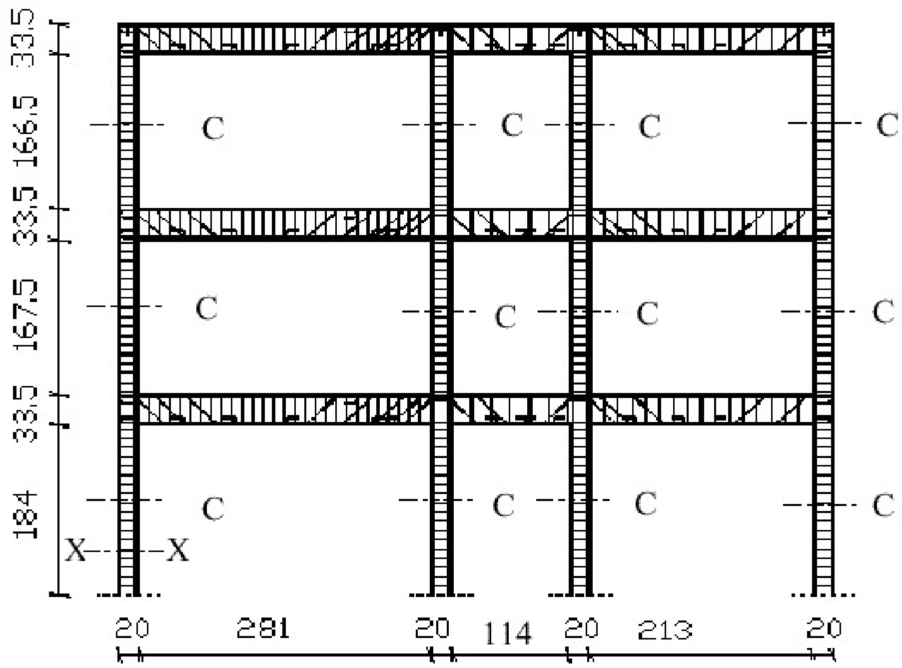

Fig. 1: Geometry and reinforcement of the frame

This frame (a 2/3 scale model) has been designed and built at the University of Pavia, according to "old" building codes, less severe for what regards reinforcement details and bars anchorage. Cyclic loading testing (performed at the University of Pavia) showed that failure occurred at the joints due to insufficient transverse reinforcement.

The following data have been assumed for the material properties (see [7]):

- Concrete:

  Maximum compressive strength:
  - $f_c = 13.3\ \mathrm{MPa}$, for all the elements, except 1st floor columns;
  - $f_c = 17.83\ \mathrm{MPa}$, for the 1st floor columns.

  Deformation corresponding to the maximum compressive strength: $\varepsilon_c = 0.002$.

  Ultimate Compressive strength: $f_{cu} = 0.6 \cdot f_c$.

  Deformation corresponding to the ultimate compressive strength: $\varepsilon_{cu} = 0.0035$.
- Steel:

  Yield strength:
  - $f_y = 385.6\ \mathrm{MPa}$, for Ø8 bars.
  - $f_y = 345.9\ \mathrm{MPa}$, for Ø12 bars.

  Initial elastic modulus: $E = 160\ \mathrm{GPa}$.

The initial fundamental period T_f of the uncracked structure is equal to 0.41 sec; the damping ratio ξ has been assumed equal to 5%.

Three are the failure mechanisms considered for the structure: 1) failure due to joints' shear stress, 2) failure due to columns' shear force, and 3) failure due to interstorey drift.

For each one of the above mechanisms, a corresponding DM parameter has been considered: 1) the joints' shear stress τ, 2) the columns' shear force T, and 3) the columns' chord rotation θ [7].

As for the IM parameter related to the seismic event, the latter has been defined as the spectral acceleration value s_a corresponding to the first natural period T_f and the damping ratio ξ of the structure. The choice is justifiable, if one considers that the structure of the example has a dominant first mode of vibration.

The critical elements chosen are the joints and columns of the frame.

Uncertainty has been implemented in hazard, demand and capacity.

The assessment procedure is divided in four steps:

- Evaluation of the hazard function for the specific site of the structure;
- Evaluation of the median demand and dispersion, for each critical element and failure mechanism considered, through a certain number of non-linear dynamic analyses;
- Evaluation of the median capacity and dispersion, for each critical element and failure mechanism considered;
- Evaluation of the seismic risk.

3.1. The Hazard

The general expression for the hazard function seen in §2 becomes in our example:

$$H_{S_a}(s_a) = P(S_a \geq s_a) = k_0 \cdot s_a^{-k} \cdot \varepsilon_{H,U} \,.$$

The parameters k and k_0 of the above relation must be calculated by regression on the **P**robabilistic **S**eismic **H**azard **A**nalysis results (see appendix II).

In this example, the structure is located near Benevento, Italy (see fig.2). In order to perform the P.S.H.A., one needs the Attenuation Law and the Gutenberg - Richter Law parameters. The latter have been evaluated on the basis of the Italian Catalogue of Earthquakes for all the adjacent influential areas while the former have been taken from [8].

The best fit values for k and k_0 obtained are: k_0=0.20863, k=2.0348 (see fig.3). Dispersion $\beta_{H,U}$ (due to uncertainty in the Guttenberg-Richter and Attenuation Laws parameters, geometry of the seismic zones etc.) has been assumed equal to 0.1.

3.2. The Demand

The procedure consists in performing a certain number of dynamic analyses in the structure and then collecting the maximum response values $\widetilde{D}$ for the DM parameters considered. The $\hat{D}$ - s_a relations are obtained by regression on the $\widetilde{D}$ values, through the range of the spectral acceleration values. Dispersion due to the seismic input randomness is then calculated.

The earthquake records selected, in order to perform the non-linear dynamic analyses, approximately evenly distributed in the s_a range 0–1.4g, can be seen in table 1 (PEER database, soil class types A and B, magnitude range between 5.5 and 7.5).[8] The model of the structure inserted on a non-linear dynamic analysis program with fibre discretisation[9] is shown in fig.5 (see also appendix III). A résumé of the procedure is given in fig.4.

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[7] Strictly, the ratio between column's top horizontal displacement u and column height L: $\theta \cong tg(\theta) = \dfrac{u}{L}$.

[8] Note that each record has been used only once and without scaling, just as proposed in [2].

[9] « FEAP – FEDEAS », Version: UNIX/PC 6.4a, 04/09/1998, see [21].

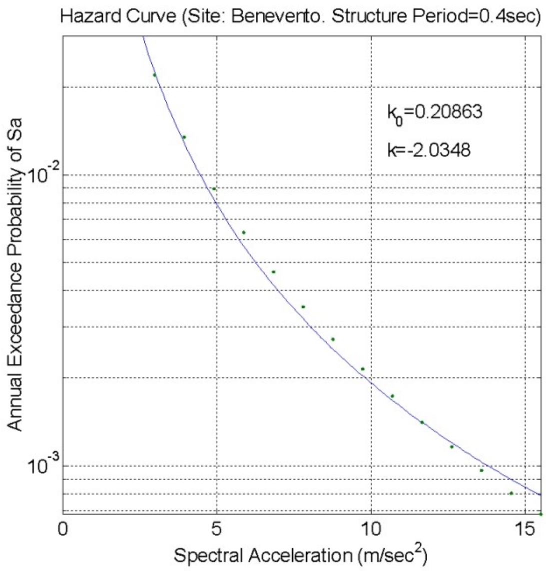


Fig.2: Map of Italy indicating the different seismic zones (N° 62: Benevento Area)

Fig.3: Seismic Hazard Curve (T_f = 0.4 sec and ξ=5%)

Earthquake Record	PGA (m/sec^2)	Sa (m/sec^2) *(T=0.417s)*	Sd (m) *(T=0.417s)*	Sa / PGA *(T=0.417s)*
FRIULI, 05/06/76, 000	2,84E-01	1,69E-01	7,50E-04	5,97E-01
PALM SPRINGS 07/08/86 360	1,01E+00	7,03E-01	3,11E-03	6,98E-01
KOBE 01/16/95 2046, TOT, 000	2,24E+00	9,87E-01	4,37E-03	4,40E-01
COALINGA 05/02/83 2342, 090	1,07E+00	1,15E+00	5,09E-03	1,07E+00
LOMA PRIETA 10/18/89 00:05, 090	8,59E-01	1,77E+00	7,85E-03	2,06E+00
NORTHRIDGE EQ 1/17/94, 12:31	1,27E+00	2,42E+00	1,07E-02	1,90E+00
CAPE MENDOCINO 04/25/92, 000	1,32E+00	2,78E+00	1,23E-02	2,11E+00
COALINGA 05/02/83 2342, 110	1,48E+00	3,17E+00	1,40E-02	2,14E+00
COALINGA 05/02/83 2342, 000	1,95E+00	3,35E+00	1,48E-02	1,71E+00
KOBE 01/16/95 2046, 090	1,65E+00	4,12E+00	1,82E-02	2,50E+00
LOMA PRIETA 10/18/89 00:05, 180	1,66E+00	4,39E+00	1,94E-02	2,64E+00
COALINGA 05/02/83 2342, 090	1,72E+00	4,98E+00	2,21E-02	2,90E+00
FRIULI, ITALY 05/06/76 2000, 000	3,45E+00	6,60E+00	2,92E-02	1,91E+00
MORGAN HILL 04/24/84,345	5,76E+00	6,73E+00	2,98E-02	1,17E+00
ERZIKAN 03/13/92 1719	4,86E+00	7,50E+00	3,32E-02	1,54E+00
ERZIKAN 03/13/92 1719	5,05E+00	7,54E+00	3,34E-02	1,49E+00
CAPE MENDOCINO 04/25/92,090	1,02E+01	7,78E+00	3,44E-02	7,63E-01
KOBE 01/16/95 2046, TOT, 090	2,72E+00	7,78E+00	3,44E-02	2,86E+00
WHITTIER NARROWS 10/01/87,000	4,40E+00	7,92E+00	3,50E-02	1,80E+00
FRIULI, 05/06/76 2000, 270	3,09E+00	8,54E+00	3,78E-02	2,77E+00
WHITTIER NARROWS 10/01/87,090	6,31E+00	8,75E+00	3,87E-02	1,39E+00
COALINGA 05/02/83 2342	5,40E+00	9,06E+00	4,01E-02	1,68E+00
GAZLI 5/17/76, KARAKYR, 000	5,97E+00	9,51E+00	4,21E-02	1,59E+00
LOMA PRIETA 10/18/89 00:05, 090	6,09E+00	9,54E+00	4,22E-02	1,57E+00
CHALFANT VALLEY 07/21/86,270	4,38E+00	1,36E+01	6,01E-02	3,10E+00

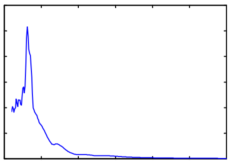
Table 1: The selected earthquake accelerogram records (PGA: Peak Ground Acceleration, S_a: Spectral Acceleration, S_d: Spectral Displacement)

Earthquake Selection
Structure Modelisation

LOOP on Earthquake Record k

Spectrum

$S_{a(k)}$

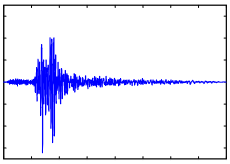


T

$\rightarrow S_{a(k)}(T = 0.41s, \xi = 5\%)$

$\equiv s_{a(k)}$

a_g

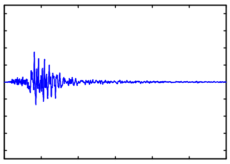


t

N-L Dynamic Analysis

LOOP on failure mechanism j considered
LOOP on critical element i
- Structural Response

$DM_{k,i,j}$



t

$\rightarrow \max(\mathrm{abs}(DM_{k,i,j}))$

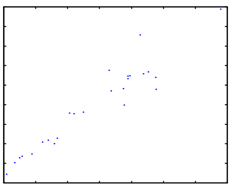
$\equiv \tilde{D}_{k,i,j}$

END of LOOP
END of LOOP

END of LOOP

LOOP on failure mechanism j considered
LOOP on critical element i
- Median Demand

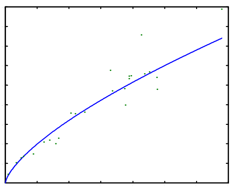
$\tilde{D}_{all\,k,i,j}$



$s_{a(all\,k)}$

Regression

$\hat{D}_{i,j}$



s_a

- Demand Dispersion due to Seismic Input Randomness

$\beta_{D_{i,j},R} \cong \sigma_{\ln(D_{i,j},R)}$

END of LOOP
END of LOOP

Fig.4: Recapitulating scheme of the procedure steps, for evaluating the median demands and corresponding dispersions for every critical element and mechanism of failure considered

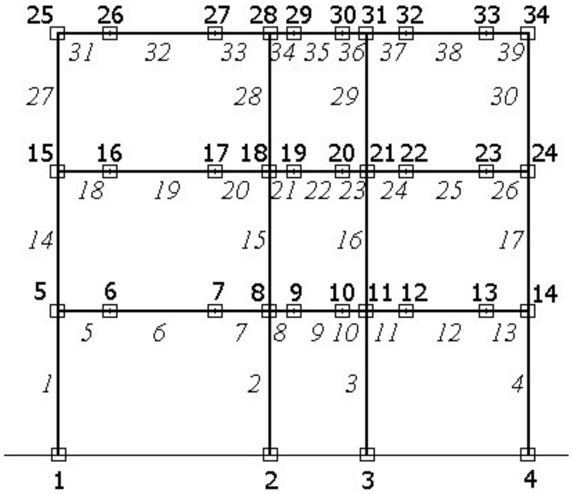


Fig.5: Structure Modelisation
(In straight characters the Joints, in italic characters the Beams & Columns)

3.2.1. Joint Shear Stress Demand D_τ

3.2.1.1. Median Value $\hat{\mathrm{D}}_\tau$

The $\hat{\mathrm{D}}_\tau$ - s_a relations obtained through regression on the dynamic analyses results are shown in fig 6.

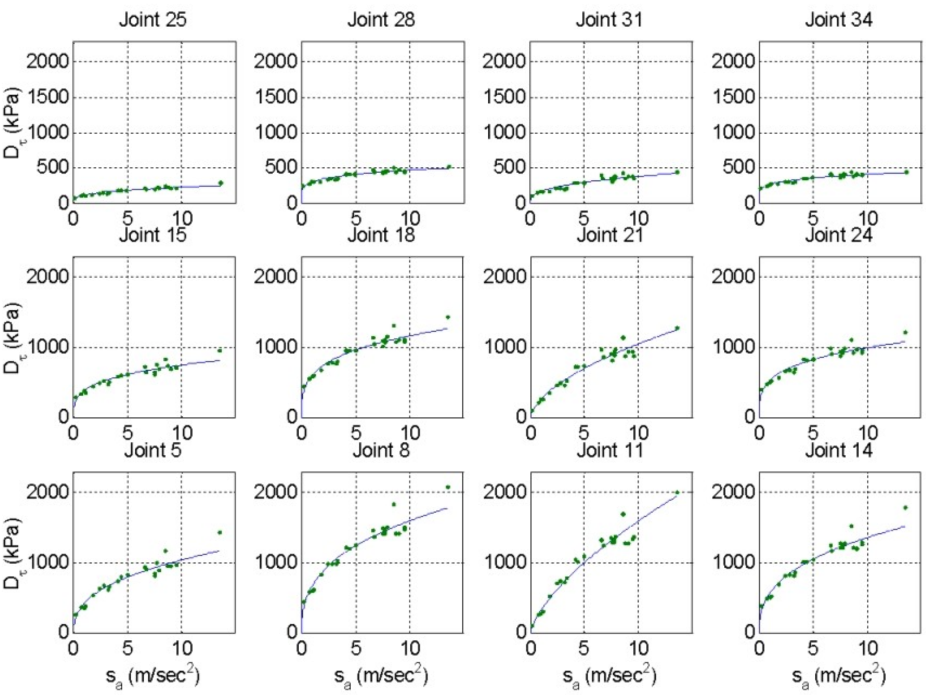


Fig.6: Median Shear Stress Demands $\hat{D}_\tau$ on the joints

3.2.1.2. Dispersion due to Randomness $\beta_{D_\tau,R}$

The dispersion on the demand β_{D_τ,R_1}, due to seismic input randomness, has been assumed equal to the standard deviation of the natural logarithm of D_τ.[10] It has been calculated for distinct spectral acceleration value ranges[11]. The results[12] can be seen in fig.7.

Dispersion β_{D_τ,R_2}, due to other sources of randomness (for example mechanical and geometrical parameters (f_c, f_y, E, A, …) randomness), has not been implemented.

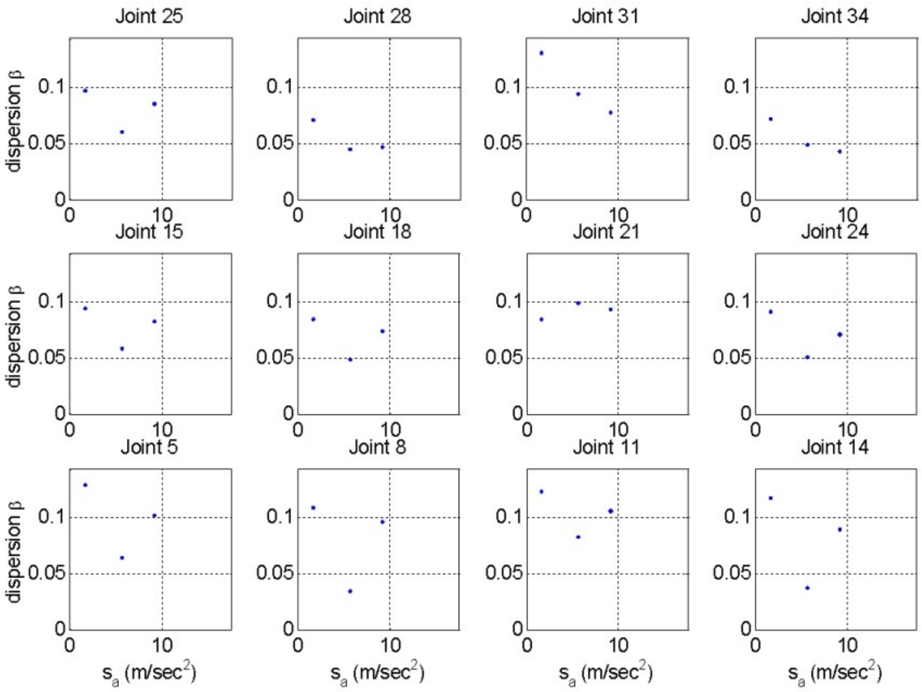


Fig.7: Dispersions $\beta_{D_\tau,R}$ on the demands

3.2.1.3. Dispersion due to Uncertainty $\beta_{D_\tau,U}$

Dispersion due to uncertainty (for example because of the assumptions on the modelisation of the structure, the lack of precision on the finite element program results etc.) has been assumed equal to 0.05.

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[10] The dispersion β of a random variable X is usually defined as the COV (coefficient of variation) of X, i.e. the ratio between the mean value E(X) and the standard deviation σ_X. In the case of a log-normally distributed variable, the standard deviation of the natural logarithm of X is approximately equal to the COV of X, when this is low (say <0.3).

[11] For instance, for a certain s_a values range m, dispersion β_m has been calculated as:

$$\beta_m = \sqrt{\frac{\sum_{k=1}^{n_m}\left[\left(\ln\left(\tilde{D}_k\right)\right)-\left(\ln\left(a \cdot s_{a(k)}{}^{b}\right)\right)\right]^2}{n_m}},$$

where index k refers to the k-earthquake record included in the m-range of size n_m. (The ranges have been selected in that way, so that an equal number of records is included to each one of them: 0-3.2, 3.2-7.6, 7.6-14 m/sec^2 (see table 1)). In this example, results have shown that the influence of the number of ranges m on the final value P_f is minor: even if the median dispersion is used (which means m=1), the difference in the P_f value is negligible.

[12] One will note that for the low values range of the spectral acceleration, the corresponding dispersions are high; this result is partially due to the fact that the demand values corresponding to s_a = 0 (static load only) are not null, as suggested by the relation $\hat{D}_\tau = a \cdot s_a^b$.

3.2.2. Column Shear Force Demand D_T & Interstorey Drift Demand D_θ

The same procedure as the one shown in §3.2.1 has been followed for the other two failure mechanisms. The corresponding $\hat{D}_T - s_a$ and $\hat{D}_\theta - s_a$ relations are shown in fig. 9 and 10.

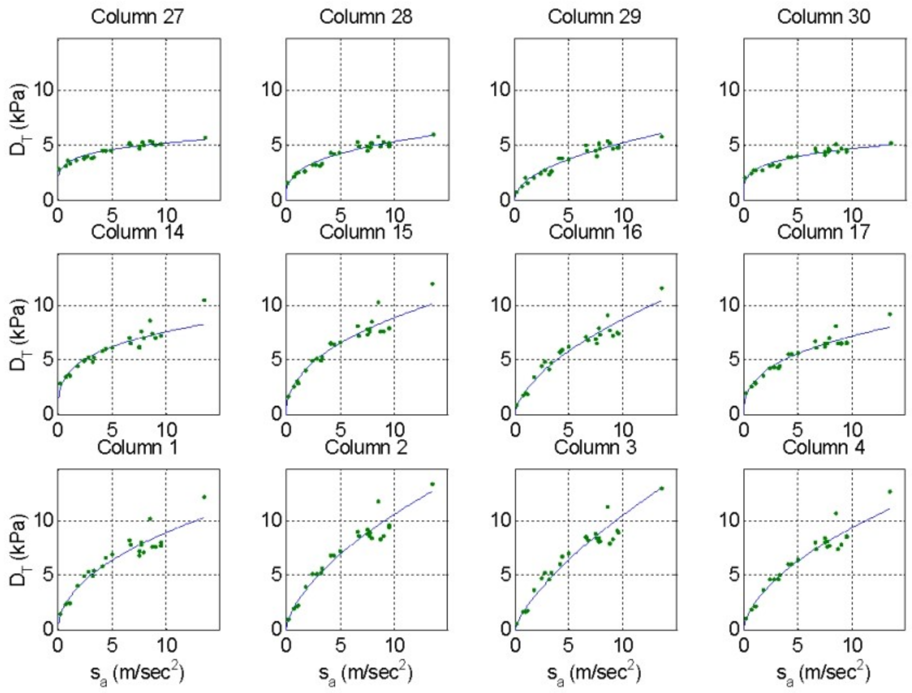

Fig.9: Median Shear Force Demands $\hat{D}_T$ on the columns

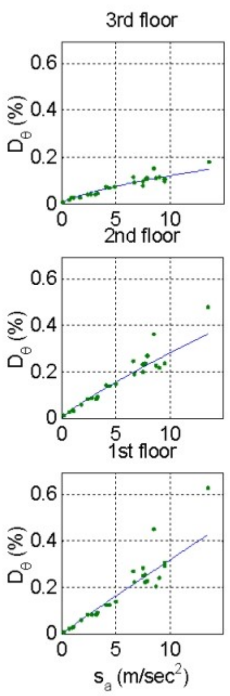

Fig.10: Median Interstorey Drift Demands $\hat{D}_\theta$ on one series of columns

3.2.a. Verification of the lognormal distribution assumption for the demand

Let's consider a critical element i, for which the median demand $\hat{D}_{i,j}$ and the demand dispersion $\beta_{D_{i,j},R}$ for the DM parameter associated to the failure mechanism j, have been evaluated. The scope of the present sub-paragraph is to control, for a certain confidence level $1-\alpha$, if the hypothesis that the demand is log-normally distributed, as originally assumed, can be affirmed.[13] For that purpose, the Experimental Cumulative Frequency (ECF) $\tilde{F}(z_{k,i,j})$ of the discrete variable $z_{k,i,j} = \dfrac{\ln(\tilde{D}_{k,i,j}) - \ln(a_{i,j} \cdot [s_a = s_{a(k)}]^{b_{i,j}})}{\beta_{D_{i,j}|s_a=s_{a(k)},R}}$ ($z_{k,i,j}$ is the k[th] largest value in the earthquake records sample of size n)[14] will be compared to the Cumulative Distribution Function (CDF) $F_Z(z_{k,i,j})$ of the Standard Normal Variable z.[15]

The goodness-of-fit test chosen is the Kolmogorov-Smirnoff one: the so-called null-hypothesis $H_{0\,(i,j)}$ (i.e. that the demand is distributed log-normally for the critical element i and the failure mechanism j) is verified if the largest of the values of the n differences between the ECF and the CDF D_2, does not exceed a critical value *c*, depending on the size of the sample n and the confidence level $1-\alpha$ (see [15]):

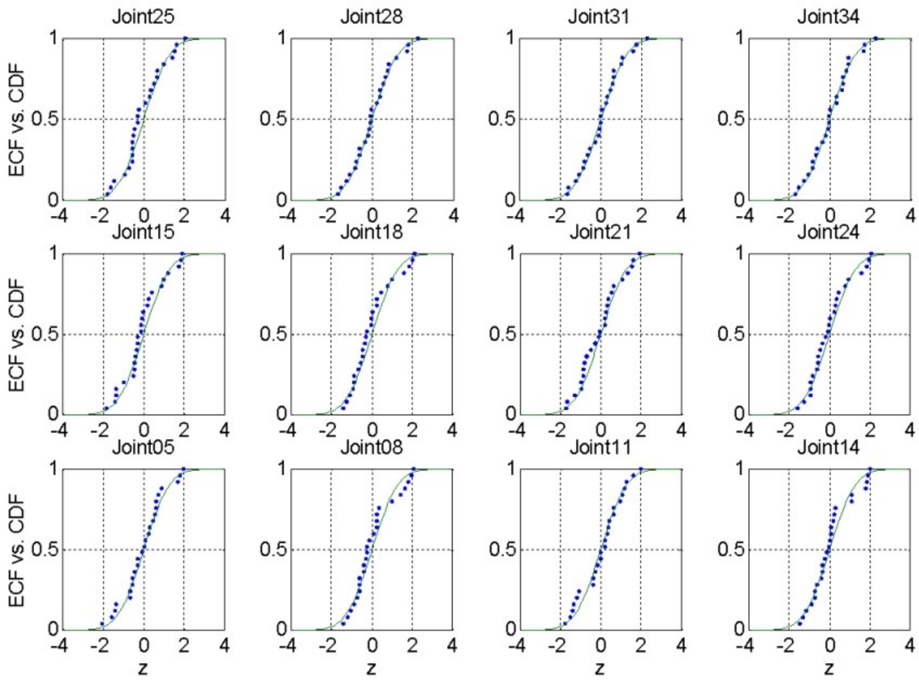

Fig.11: Comparison between ECF and CDF for the case of joint shear stress demand

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[13] In our case, α is the probability to reject the log-normality of the distribution, when in fact the hypothesis is true. Note that when the confidence level 1-α increases, the probability β to accept the log-normality of the distribution, when in fact the hypothesis is wrong, increases also!

[14] The Experimental Cumulative Frequency $\tilde{F}(z_{k,i,j})$ is equal to $\dfrac{k}{n}$.

[15] The Cumulative Distribution Function of z is equal to $P(z < Z) = \int_{-\infty}^{Z} \frac{1}{\sqrt{2\cdot\pi}} \cdot \exp\left(-\frac{z^2}{2}\right) \cdot dz$ ·

$$\text{Accept } H_0 \text{ if } D_2 \le c,$$
$$\text{where } D_{2\,(i,j)} = \max\left\{ \left| \widetilde{F}\left(z_{k,i,j}\right) - F_Z\left(z_{k,i,j}\right) \right| \right\}.$$

3.2.a.1. Case of Joint Shear Stress Demand

The values D_2 of the Kolmogorov-Smirnoff test have been evaluated for the joint shear stress demands calculated on §3.2.1. Results can be seen in Table 2 and fig.11.

Joints	5	8	11	14	15	18	21	24	25	28	31	34
D_2	*0.071*	*0.118*	*0.098*	*0.14*	*0.157*	*0.123*	*0.123*	*0.095*	*0.162*	*0.079*	*0.066*	*0.056*

Table 2: D_2 results in the case of Joint Shear Stress

The critical value c, corresponding to an earthquake sample of size 25 and a confidence level $1-\alpha$ of 95%, is equal to 0.27. The D_2 values seen in table 2 are lower than the value c; therefore the original assumption, that the shear stress demand on the joints is distributed log-normally, can be accepted, for the specified confidence level.

3.3. The Capacity

For each DM parameter considered, the median value of the capacity, corresponding to the ultimate limit state, has been taken from the literature.

3.3.1. Joint Shear Stress Capacity C_τ

The median shear stress capacity, corresponding to the joint cracking, is given by the semi-empirical relation below ([9]):

$$C_\tau = 0.29 \cdot \sqrt{f_c}, \qquad (f_c \text{ in MPa})$$

Randomness on the above expression derives from the aleatory character of f_c, considered as a random variable, log-normally distributed.[16]

The dispersion on the capacity $\beta_{C_\tau,R}$, as equal to the dispersion of the square root of f_c, depends on the dispersion of f_c itself. The latter is given equal to 0.15 (see [10]); the former has thus been calculated equal to 0.075.[17]

Dispersion due to uncertainty (for example on the above expression and on the PDF parameters of f_c) is assumed equal to 0.05.

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[16] Note that if f_c is considered log-normally distributed, its square root is not log-normally distributed! The approximation can be however considered acceptable.

[17] If X is a random variable with median $\hat{X}$ and standard deviation σ_X, the median $\hat{Y}$ and standard deviation σ_Y of the random variable $Y = f(X)$ can be evaluated by the following formulas (see [15]):

$$\hat{Y} = f\left(\hat{X}\right), \quad \sigma^2{}_Y = \sigma^2{}_X \cdot \left(\frac{dY}{dX}\right)^2 \Big|_{Y \approx \hat{Y}}.$$

The dispersion $\beta_{C_\tau,U}$ can be represented by the coefficient of variation of C_τ:

$$\beta_{C_\tau,U} = \frac{\sigma_{C_\tau}}{\hat{C}_\tau} = \frac{\sigma_{f_c} \cdot \frac{dC_\tau}{df_c}\Big|_{C\tau=\hat{C}\tau}}{0.29 \cdot \sqrt{\hat{f}_c}} = \frac{\sigma_{f_c} \cdot \frac{0.29}{2 \cdot \sqrt{\hat{f}_c}}}{0.29 \cdot \sqrt{\hat{f}_c}} = \frac{\sigma_{f_c}}{2 \cdot \hat{f}_c} = \frac{\beta_{f_c,U}}{2} = 0.075$$

3.3.2. Column Shear Force Capacity C_T

The median Shear Force Capacity relation has been taken from [10]:

$$C_T = V_{R3} = \frac{0.9 \cdot d}{s} \cdot A_{sw} \cdot f_{yw} + \left[\tau_R \cdot k \cdot \left(1.2 + 40 \cdot \frac{A_s}{b \cdot d} \right) + 0.15 \cdot \sigma \right] \cdot b \cdot d \cdot \beta ,$$

where A_s and A_{sw} are the longitudinal and transverse reinforcement steel bar section respectively, f_{yw} the yield strength of the transverse reinforcement bars, s the stirrup spacing, τ_R the concrete shear strength, b the width and d the static height of the concrete section, k a scale coefficient equal to $1.6 - d \geq 1$ (d in m.), $\sigma = N/_{b \cdot h}$ the normal stress in the section resulting from axial compression and β a coefficient taking into account the concentrated loads at the vicinity of the support.

3.3.3. Interstorey Drift Capacity C_θ

The median Interstorey Drift Capacity has been taken from [11]:

$$C_\theta = \alpha_{st,w} \cdot (1 - 0.38 \cdot a_{cyc}) \cdot \left(1 + \frac{a_{sl}}{1.7} \right) \cdot (1 - 0.375 \cdot a_{wall}) \cdot (0.3^{\nu}) \cdot \left(\frac{\max(0.01, \omega_2)}{\max(0.01, \omega_1)} \cdot f_c \right)^{0.2} \cdot \left(\frac{L_s}{h} \right)^{0.425} \cdot 25^{\alpha \cdot \rho_{sx} \cdot \frac{f_{ywd}}{\hat{f}_c}} \cdot 1.45^{100 \rho_d} ,$$

where L_s/h is the shear span ratio, ω_1 and ω_2 the mechanical ratios of tension and compression reinforcement respectively, $\nu = N/bhf_c$ the normalised axial force (b and h the dimensions of the section), $\rho_{sx} = A_{sw}/bs$ the geometric ratio of transverse reinforcement (s the stirrup spacing), ρ_d the geometric ratio of bi-diagonal reinforcement, $\alpha_{st,w}$ a coefficient for steel equal to 0.016 for ductile steels (S400 or Tempcore type) and 0.0105 for brittle steels, a_{cyc} a coefficient relative to the nature of loading, equal to 0 for monotonic and 1 for cyclic loading, a_{sl} a coefficient relative to the bars slippage, equal to 1 if is expected, 0 otherwise, and a_{wall} a coefficient equal to 1 if the element is a wall and 0 otherwise.[18]

3.4. The Risk

In order to evaluate the total seismic risk P_f of the structure, two parts have been considered:

- First, the median partial annual probabilities $\hat{p}_j$ of failure, due to the different mechanisms j, and corresponding dispersions due to uncertainty have been calculated:

$$\hat{p}_j = 1 - \prod_i (1 - \hat{p}_{i,j})$$

$$\beta_{p_j,U} \cong \hat{p}_j \cdot \left(\sqrt{\sum_{i=1}^{N_e} \frac{\hat{p}^2_{i,j}}{\beta^2_{p_{i,j},U}}} \right)^{-1} ,$$

[18] One will note in the above expressions the dependence between capacity C, related to the axial load N, and demand D. The dependence can be removed, by considering the normalised DM parameter D/C; the probability of failure is then given by the expression P[D/C≥1]).

where $\hat{p}_{i,j}$ is the median probability of failure of the i-element, due to the j-mechanism, and $\beta_{p_{i,j},U}$ the corresponding dispersion (due to uncertainty):

$$\hat{p}_{i,j} = \hat{H}_{S_a}\left(s_a(\hat{C}_{i,j})\right)\cdot \exp\left(\frac{k^2}{2\cdot b^2}\left(\beta^2_{D_{i,j},R|s_a(\hat{C}_{i,j})} + \beta^2_{C_{i,j},R}\right)\right)$$

$$\beta_{p_{i,j},U} = \sqrt{\frac{k^2}{b^2}\cdot\beta^2{}_{D_{i,j},U} + \frac{k^2}{b^2}\cdot\beta^2{}_{C_{i,j},U} + \beta^2{}_{H,U}}\ .$$

- Then, the median seismic risk and corresponding dispersion (due to uncertainty) has been evaluated:

$$\hat{P}_f = 1 - \prod_j (1 - \hat{p}_j)$$

$$\beta_{P_f,U} \cong \hat{P}_f \cdot \left(\sqrt{\sum_{j=1}^{N_m} \frac{\hat{p}^2{}_j}{\beta^2{}_{p_j,U}}}\right)^{-1}.$$

3.4.1. Probability of joints' failure $\hat{p}_\tau$ due to Shear Stress

Table 3 recapitulates all the results found until now (a and b demand parameters, median capacities, dispersions on the demands and the capacities). These results permit us to evaluate the probability of failure for each joint $p_{i,\tau}$ due to the shear stress.

Note that the probability of failure due to shear stress has been evaluated only for those joints whose spectral acceleration producing the median capacity $s_a(\hat{C}_\tau)$ does not exceed the arbitrarily chosen value of 1.6 g. (In fact, from the hazard curve of fig.3, one sees that an earthquake producing a spectral acceleration equal to 1.6g has an annual exceedance probability equal to $7\cdot 10^{-4}$).

Joints	5	8	11	14	15	18	21	24	25	28	31	34
a	*429.1*	*629.4*	*343.3*	*581.9*	*392.2*	*616.7*	*273.9*	*538.1*	*117.5*	*302.1*	*168.2*	*268.5*
b	*0.384*	*0.362*	*0.665*	*0.367*	*0.28*	*0.275*	*0.582*	*0.267*	*0.285*	*0.189*	*0.356*	*0.183*
$\hat{C}_\tau$	*1144*	*1144*	*1144*	*1144*	*1057*	*1057*	*1057*	*1057*	*1057*	*1057*	*1057*	*1057*
$s_a(\hat{C}_\tau)$	*12.84*	*4.002*	*6.108*	*6.303*	*34.46*	*7.086*	*10.18*	*12.59*	*2202*	*739*	*173*	*1785*
$\beta_{D_\tau,R\|s_a(\hat{C}_\tau)}$	*0.102*	*0.035*	*0.083*	*0.037*		*0.049*	*0.094*	*0.071*				
$\beta_{D_\tau,U}$	*0.05*	*0.05*	*0.05*	*0.05*		*0.05*	*0.05*	*0.05*				
$\beta_{C_\tau,R}$	*0.075*	*0.075*	*0.075*	*0.075*		*0.075*	*0.075*	*0.075*				
$\beta_{C_\tau,U}$	*0.05*	*0.05*	*0.05*	*0.05*		*0.05*	*0.05*	*0.05*				
$\hat{p}_{i,\tau}$ (%)	*0.145*	*1.383*	*0.557*	*0.549*		*0.484*	*0.203*	*0.165*				
$\beta_{p_\tau,U}$	*0.387*	*0.41*	*0.238*	*0.404*		*0.531*	*0.267*	*0.548*				

Table 3: Evaluation of the probability of failure for each critical joint due to shear stress ($\hat{D}_\tau = a\cdot s_a{}^b$ the Median Demand (kPa), $\hat{C}_\tau$ the Median Capacity (kPa), $s_a(\hat{C}_\tau)$ the Spectral Acceleration value producing the Median Capacity (m/sec^2), $\beta_{D_\tau,R|s_a(\hat{C}_\tau)}$ and $\beta_{D_\tau,U}$ the Dispersions on the Demand due to randomness and uncertainty respectively (the first one calculated at the vicinity of the Spectral Acceleration value producing the Median Capacity), $\beta_{C_\tau,R}$ and $\beta_{C_\tau,U}$ the dispersions due to randomness and uncertainty respectively on the Capacity, $\hat{p}_{i,\tau}$ the median probability of failure for each joint and $\beta_{p_\tau,U}$ the corresponding dispersion due to uncertainty)

The median annual probability of failure $\hat{p}_\tau$ due to joint shear stress is equal to 3.44%. Its corresponding dispersion β_{p_τ} is equal to 0.76.

3.4.2. Probabilities of columns failure $\hat{p}_T$ and $\hat{p}_\theta$ due to Shear Force & Interstorey Drift

Tables 4 and 5 recapitulate the results found until now. One can understand easily that the probabilities of columns' failure due to shear and interstorey drift are very low and surely negligible if compared to the probability of joints failure, since the spectral acceleration values producing the median capacity for the DM parameters considered are very high.

Columns	1	2	3	4	14	15	16	17	27	28	29	30
a	*2.942*	*2.59*	*2.076*	*2.513*	*3.88*	*3.21*	*2.264*	*3.106*	*3.46*	*2.49*	*1.686*	*2.714*
b	*0.479*	*0.609*	*0.702*	*0.568*	*0.29*	*0.439*	*0.584*	*0.362*	*0.177*	*0.33*	*0.491*	*0.237*
$\hat{C}_T$	*27.2*	*30.83*	*29.51*	*26.61*	*21.03*	*23.29*	*22.61*	*20.74*	*18.99*	*19.97*	*19.85*	*18.99*
$s_a(\hat{C}_T)$	*104.1*	*58.45*	*43.92*	*63.78*	*343.3*	*91.14*	*51.28*	*189.6*	10^4	*552*	*151.8*	*3636*

Table 4: s_a values producing median shear capacity in the columns ($\hat{D}_T = a \cdot s_a{}^b$ the Median Demand (kN), $\hat{C}_T$ the Median Capacity (kN), $s_a(\hat{C}_T)$ the Spectral Acceleration value producing the Median Capacity (m/sec^2))

Columns	1	14	27
a	*0.034*	*0.039*	*0.024*
b	*0.968*	*0.85*	*0.685*
$\hat{C}_\theta$	*5.919*	*5.699*	*5.765*
$s_a(\hat{C}_\theta)$	*206.2*	*346.9*	*2844*

Table 5: s_a values producing median interstorey drift capacity in one series of columns ($\hat{D}_\theta = a \cdot s_a{}^b$ the Median Demand (%), $\hat{C}_\theta$ the Median Capacity (%), $s_a(\hat{C}_\theta)$ the Spectral Acceleration value producing the Median Capacity (m/sec^2))

3.4.3. Seismic Risk of the structure $\hat{P}_f$

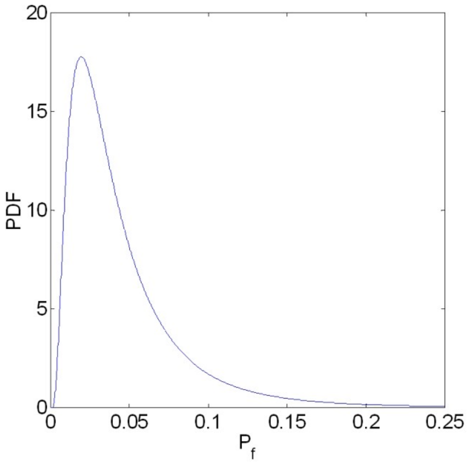

Fig.12: Probability Density Function of the Seismic Risk

Recapitulating the results found on the two previous sub-paragraphs, one can conclude that the total annual probability of the structure's failure, due to earthquake solicitation, is equal to the annual probability of joints failure due to shear cracking. (The median value $\hat{P}_f$ of the probability is equal to 3.44% and its dispersion β_{P_f} is equal to 0.76, see fig.12).

3.4.a. Evaluation of the Minimum Number of NL Dynamic Analyses required

As seen in §3.2, the number of non-linear dynamic analyses performed, in order to arrive at the final result of §3.4, has been equal to 25. However, a lower number of dynamic analyses could result in a final outcome not different from the one found.

In order to estimate the minimum number of analyses required, the following procedure has been conceived:

Let's consider a random rearrangement m_i of the earthquake records table (see t.1). The sample $m_{i,j}$ contains the first j elements of m_i. For each sample $m_{i,j}$, the median seismic risk $\hat{P}_f$ of the structure is computed.

The results, for i = 100 and j_{min} = 8, are shown in fig. 13.

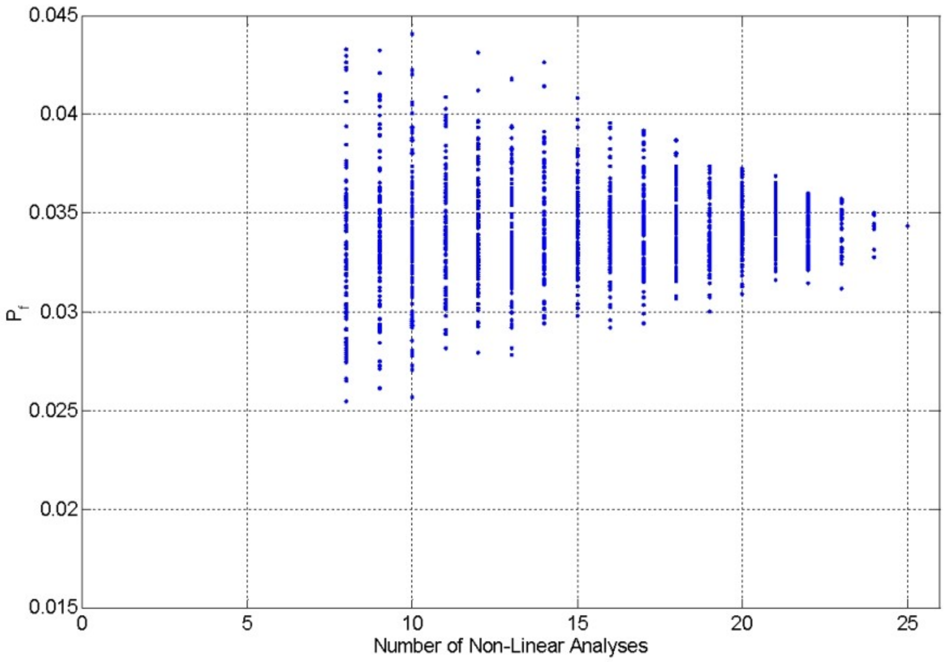

Fig.13: $\hat{P}_f$ as a function of the number of non-linear dynamic analyses performed

One can see that even for a minimum number of analyses, say 8, the order of magnitude for $\hat{P}_f$ remains the same.

Figure 14 shows the Confidence Intervals for the mean value of $\hat{P}_f$, as a function of the number of non-linear dynamic analyses performed, corresponding to two significance levels: 5 and 1%.[19]

[19] One of the basic problems of statistic is to estimate the parameters of a population, given a seldom sample. If one adopts the mean of the sample μ as an estimator of the mean of the population X, he

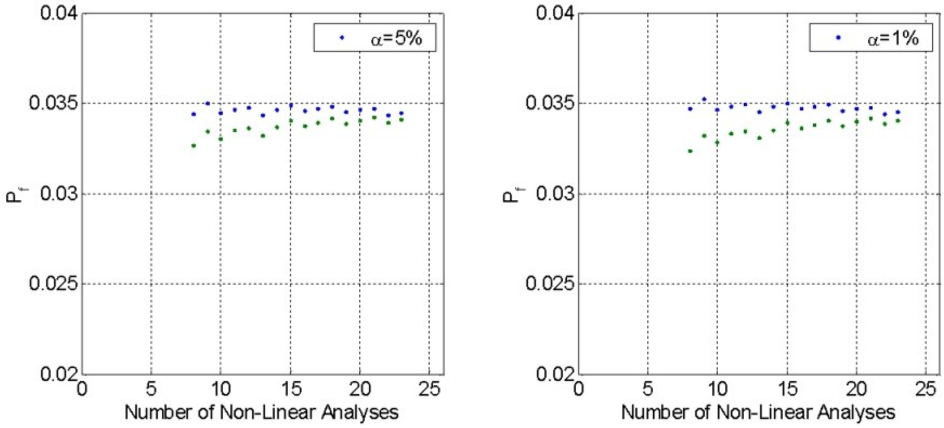

Fig.14: Confidence Intervals for the mean value of $\hat{P}_f$

4. Conclusions

The example shows the applicability of the SAC/FEMA 2000 Seismic Risk Assessment Approach to reinforced concrete structures, by selection of appropriate mechanisms of failure.

The final outcome (i.e. elevated probability of joints failure due to shear stress) agrees with the deterministic analysis predictions and results of the tests performed by the University of Pavia.

The main advantage of the method remains the drastic reduction of the number of non-linear dynamic analyses performed (if one compares with classical simulation methods such as the Monte Carlo one), considering moderate simplifications.

One must not forget that the choice of the spectral acceleration as an IM parameter related to the seismic event exclude structures whose first mode of vibration is not a dominant one. One can always choose an IM parameter related to the ground acceleration, such as the PGA, but the use of the spectral acceleration parameter has shown minor dispersion on the results.

Future research can concentrate its focus on inserting other sources of randomness, apart the seismic input one, in the demand quantity (for example randomness due to the aleatory character of the mechanical and geometrical parameters related to the structure).

Actual research reports on the SAC/FEMA 2000 Seismic Risk Assessment Approach can be consulted (see [16], [17]).

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must also define an interval in which he can be sure, at a confidence level 1-α, that the mean X is included. The interval is called Confidence Interval (CI) and is dependent on the standard deviation σ and the size n of the sample.

$$P[(\mu-\varepsilon) < X < (\mu+\varepsilon)] = 1-\alpha,$$

where (μ-ε, μ+ε) is the Confidence Interval. (The precision ε is equal to $z_{1-\frac{\alpha}{2}} \cdot \frac{\sigma}{\sqrt{n}}$, for the case of high size samples (n over 30), where $z_{1-\frac{\alpha}{2}}$ is the Standard Normal Variable value corresponding to a CDF value of $1-\frac{\alpha}{2}$).

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References

[1] Cornell C.A., Vamvatsikos D., Jalayer F. and Luco N.: *Seismic Reliability of Steel Structures.* Proc. of 9th IFIP WG 7.5 Working Conference on Reliability & Optimization of Structural Systems, September 2000, Ann Arbor, MI.

[2] Jalayer F., Cornell C.A.: *Development of a probability-based demand and capacity factor design seismic format.* PEER Report 2000. Publication pending.

[3] Cornell, C.A.: *A probability and Non-Linear Displacement Based Seismic Format: Derivation of the Demand, Capacity and Confidence Factors used in the 2000 SAC Steel-Moment-Resisting-Frame Guidelines.* Internal Technical Note, August 2000.

[4] Bazurro P., Cornell C.A.: *Seismic Hazard Analysis of Non-Linear Structures. I: Methodology.* Journal of Structural Engineering, ASCE, Vol. 120, N° 11, p.3320-3344.

[5] Bazurro P., Cornell C.A.: *Seismic Hazard Analysis of Non-Linear Structures. II: Applications.* Journal of Structural Engineering, ASCE, Vol. 120, N° 11, p.3345-3365.

[6] Cornell C.A.: *Calculating building seismic performance reliability: a basis for multilevel design norms.* Internal Technical Note, Dept. of Civil Engineering, Stanford University, CA, 1996.

[7] Calvi G.M., Pampanin S., Moratti M., Ward J., Magenes G., Pavese A., Rasulo A.: *Seismic Vulnerability assessment of existing reinforced concrete buildings, Experimental Test on a 3-Story R.C. Frame.* Internal Technical Note, University of Pavia.

[8] Fabio Sabetta & Antonio Pugliese: *Estimation of Response Spectra and Simulation of Non-Stationary Earthquake Ground Motions.* Bulletin of the Seismological Society of America, Vol. 86, No. 2, pp. 337-352, April 1996.

[9] Priestley M.J.N., Seible F., Calvi G.M.: *Seismic Design and Retrofit of Bridges.* Wiley-Interscience, 1995

[10] *Eurocode 2: Design of Concrete Structures.*

[11] Panagiotakos T., Fardis M.N.: *Deformations of Reinforced Concrete Members at Yielding and Ultimate.* Proceedings of 2nd National Conference on Earthquake Engineering (Greece). Publication pending.

[12] Pinto P.E.: *Reliability Methods in Earthquake Engineering.* Progr. Struct. Eng. Mater., Vol.3, March 2001.

[13] Chopra A.K.: *Dynamics of Structures – Theory and Applications to Earthquake Engineering.* Prentice-Hall, 1995.

[14] Kramer S.L.: *Geotechnical Earthquake Engineering.* Prentice-Hall, 1996.

[15] Ang A., Tang W.: *Probability Concepts in Engineering Planning & Design.* John Wiley & Sons, 1975.

[16] Lupoi G.: *Procedura di Affidabilità per le Strutture a Telaio in Cemento Armato. (Affidability Procedure for Reinforced Concrete Frames).* Bachelor Degree Thesis, Università di Roma La Sapienza, October 2001.

[17] Rizza R.: *Seismic Isolation of existing RC structures.* SAFERR Research Project Report. Under preparation.

APPENDICES

I. Review of the SAC/FEMA 2000 Method

The three basic assumptions of the method are:

1) Hazard Function:

$$H_{IM}(i_m) = k_0 \cdot i_m^{-k} \cdot \varepsilon_{H,U};$$

2) Demand $D_{i,j}$ and Capacity $C_{i,j}$ for the DM parameter associated to the mechanism of failure j in the element i, are considered as independent log-normally distributed random variables;
3) Median Demand Value:

$$\hat{D}_{i,j} = a_{i,j} \cdot i_m^{b_{i,j}}$$

(indices refer to the i-th critical element and the j-th mechanism of failure).

The first step is to couple the IM parameter, related to the hazard, with the demand of the DM parameter, in order to produce an (element-specific) DM parameter hazard function:

$$H_{i,j}(c_{i,j}) = P(D_{i,j} \geq c_{i,j}).$$

($H_{i,j}(c_{i,j})$ is defined as the annual probability that the capacity value $c_{i,j}$ of the DM parameter associated to the mechanism j is exceeded, for the critical element i).

Given the assumptions (2) and (3), the DM parameter hazard function becomes:

$$\begin{aligned} H_{i,j}(c_{i,j}) &= P(\hat{D}_{i,j} \cdot \varepsilon_{D_{i,j},R} \cdot \varepsilon_{D_{i,j},U} \geq c_{i,j}) \\ &= P(a_{i,j} \cdot i_m^{b_{i,j}} \cdot \varepsilon_{D_{i,j},R} \cdot \varepsilon_{D_{i,j},U} \geq c_{i,j}) \\ &= \int P(a_{i,j} \cdot i_m^{b_{i,j}} \cdot \varepsilon_{D_{i,j},R} \cdot \varepsilon_{D_{i,j},U} \geq c_{i,j}) \cdot f(\varepsilon_{D_{i,j},R}) \cdot d\varepsilon_{D_{i,j},R}, \end{aligned}$$ [20]

where $f(\varepsilon_{D_{i,j},R})$ is the probability density function of the variable $\varepsilon_{D_{i,j},R}$.

Given the assumption (1), the relation above becomes:

$$\begin{aligned} &\int P\left(i_m \geq \left(\frac{c_{i,j}}{a_{i,j} \cdot \varepsilon_{D_{i,j},R} \cdot \varepsilon_{D_{i,j},U}} \right)^{\frac{1}{b_{i,j}}} \right) \cdot f(\varepsilon_{D_{i,j},R}) \cdot d\varepsilon_{D_{i,j},R} \\ &= \int H_{IM}\left(\left(\frac{c_{i,j}}{a_{i,j} \cdot \varepsilon_{D_{i,j},R} \cdot \varepsilon_{D_{i,j},U}} \right)^{\frac{1}{b_{i,j}}} \right) \cdot f(\varepsilon_{D_{i,j},R}) \cdot d\varepsilon_{D_{i,j},R} \\ &= \int k_0 \cdot \left(\frac{c_{i,j}}{a_{i,j} \cdot \varepsilon_{D_{i,j},R} \cdot \varepsilon_{D_{i,j},U}} \right)^{\frac{-k}{b_{i,j}}} \cdot \varepsilon_{H,U} \cdot f(\varepsilon_{D_{i,j},R}) \cdot d\varepsilon_{D_{i,j},R} \end{aligned}$$

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[20] Total Probability Theorem: Given an event B conditioned on the mutually exclusive and collectively exhaustive events A_1, A_2, …, A_n, the probability P(B) of the event to happen is equal to $\sum_{i=1}^{n} P(B \mid A_i) \cdot P(A_i)$, where $P(B|A_i)$ is the probability of B to happen, if A_i has already happen. In our case, $P(a \cdot i_m^{b} \cdot \varepsilon \geq c, \varepsilon = \varepsilon) = \sum_{\varepsilon} P(a \cdot i_m^{b} \cdot \varepsilon \geq c \mid \varepsilon = \varepsilon) \cdot P(\varepsilon) \cdot$

$$= k_0 \cdot \left(\frac{c_{i,j}}{a_{i,j} \cdot \varepsilon_{D_{i,j},U}} \right)^{\frac{-k}{b_{i,j}}} \cdot \varepsilon_{H,U} \cdot \int \left(\frac{1}{\varepsilon_{D_{i,j},R}} \right)^{\frac{-k}{b_{i,j}}} \cdot f\left(\varepsilon_{D_{i,j},R}\right) \cdot d\varepsilon_{D_{i,j},R} \; .$$

The evaluation of the integral yields the final expression for the DM parameter hazard function.

$$H_{i,j}\left(c_{i,j}\right) = k_0 \cdot \left(\frac{c_{i,j}}{a_{i,j}} \right)^{\frac{-k}{b_{i,j}}} \cdot \exp\left(\frac{k^2}{2 \cdot b^2} \cdot \beta_{D_{i,j},R} \right) \cdot \varepsilon^{\frac{k}{b}}{}_{D_{i,j},U} \cdot \varepsilon_{H,U}$$

The seismic risk (for the element i and the mechanism j) is equal to:

$$p_{i,j} = P\left(D_{i,j} \geq C_{i,j}\right).$$

This expression is similar to the one giving the DM parameter Hazard Function seen above, with the difference that $C_{i,j}$ is now a random variable and not a deterministic one. The second step of the procedure consists in evaluating this expression:

$$\begin{aligned} p_{i,j} &= \int P\left(D_{i,j} \geq \hat{C}_{i,j} \cdot \varepsilon_{C_{i,j},R} \cdot \varepsilon_{C_{i,j},U}\right) \cdot f\left(\varepsilon_{C_{i,j},R}\right) \cdot d\varepsilon_{C_{i,j},R} \\ &= \int H_{i,j}\left(\hat{C}_{i,j} \cdot \varepsilon_{C_{i,j},R} \cdot \varepsilon_{C_{i,j},U}\right) \cdot f\left(\varepsilon_{C_{i,j},R}\right) \cdot d\varepsilon_{C_{i,j},R} \end{aligned}$$

The final expression for the seismic risk becomes:

$$p_{i,j} = k_0 \cdot \left(\frac{\hat{C}_{i,j}}{a_{i,j}} \right)^{\frac{-k}{b_{i,j}}} \cdot \exp\left(\frac{k^2}{2 \cdot b^2} \cdot \left(\beta^2{}_{D_{i,j},R} + \beta^2{}_{C_{i,j},R} \right) \right) \cdot \varepsilon^{\frac{k}{b}}{}_{D_{i,j},U} \cdot \varepsilon^{\frac{-k}{b}}{}_{C_{i,j},U} \cdot \varepsilon_{H,U} \, ,$$

where $\left(\frac{\hat{C}_{i,j}}{a_{i,j}} \right)^{\frac{1}{b_{i,j}}}$ is the IM parameter value producing the median capacity $\hat{C}_{i,j}$.

II. Probabilistic Seismic Hazard Analysis (P.S.H.A.)

The final outcome of a P.S.H.A. is the hazard function, i.e. the probability of exceedance of a certain IM parameter value i_m, during a particular time period (see [14]).

The procedure is divided in four steps:

- First, all seismic zones capable of producing significant ground motion at the site are identified;
- Second, the Gutenberg–Richter parameters, related to the seismicity of each zone considered, are calculated.
- Third, the attenuation law parameters for the IM parameter related to the seismic event are estimated.
- Finally, the Hazard Function is evaluated by combination of the results found on previous steps.

The scope of the present appendix is to describe in detail the steps followed in order to arrive at the results of §3.1. The basic hypotheses are reminded:

- The IM parameter adopted has been defined as the spectral acceleration S_a:

$$H_{S_a}(s_a) = P(S_a \geq s_a)\,;$$

- The time period for which the hazard function has been evaluated has been defined as the period of one year.
- The structure is located near Benevento, Italy.

II.1. Seismic zone selection

All adjacent zones in a minimum radius of 80 km have been considered (see fig.A.1).

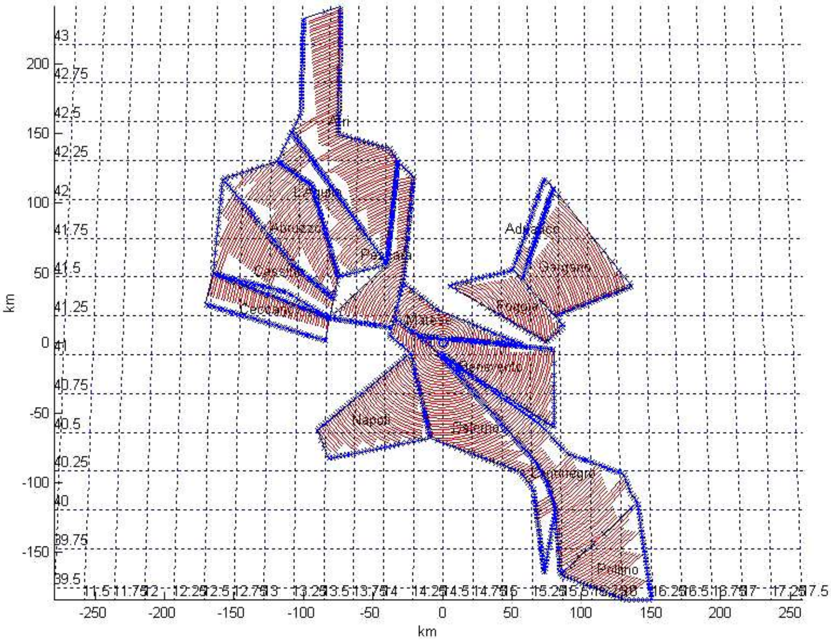

Fig.A.1: Seismic Zones Selection (see also fig.2)

II.2. Gutenberg–Richter Law Parameters Evaluation

The parameters of the Gutenberg–Richter Bounded Law have been evaluated for each seismic zone selected:

$$\nu_m = \nu_0 \cdot \frac{e^{-\beta \cdot m} - e^{-\beta \cdot m_x}}{e^{-\beta \cdot m_0} - e^{-\beta \cdot m_x}},$$

where $\nu_m = N/T$ is the mean annual frequency (if T is expressed in years) of the number of seismic events N having a magnitude greater than m, β is a parameter characterising the seismicity of the region, m_0 and m_x are the lower and higher threshold values considered for the magnitude, and ν_0 is the mean annual frequency of the number of seismic events having a magnitude greater than m_0. The limit m_0 has been chosen in order to exclude seismic events of low magnitude, that do not affect structures; m_x, on the other hand, expresses an estimation of the maximum seismic energy liberated.

The following procedure has been used for each seismic zone:

- All data relative to the zone have been taken from the Italian Catalogue of Earthquakes and have been implemented into a G-R Law Parameters Evaluation program; [21]
- A mean frequency value ν_m has been chosen, based on the Stepp diagram results,[22] for each value of magnitude m chosen;

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[21] « P.R.S. » (Procedura Rischio Sismico), E.N.E.L. – Università dell'Aquila ([18]).

[22] Let's consider a seismic event k, with magnitude m, that took place ΔT_k years ago. The Stepp diagram is constructed by reporting on a log-log scale the points (ΔT_k, $\nu_{m(k)}$), where $\nu_{m(k)}$ is the mean annual frequency of the number of earthquakes (of magnitude m) happened the last ΔT_k years ($\nu_{m(k)} = k/\Delta T_k$). If data were complete, ν_m would converge to a certain value; however, except for the last century, earthquake data are not trustworthy; that's why most Stepp diagrams present a peak (see fig. A.2.). The suggested value ν_m to be chosen is the one corresponding to this peak (see also [19]).

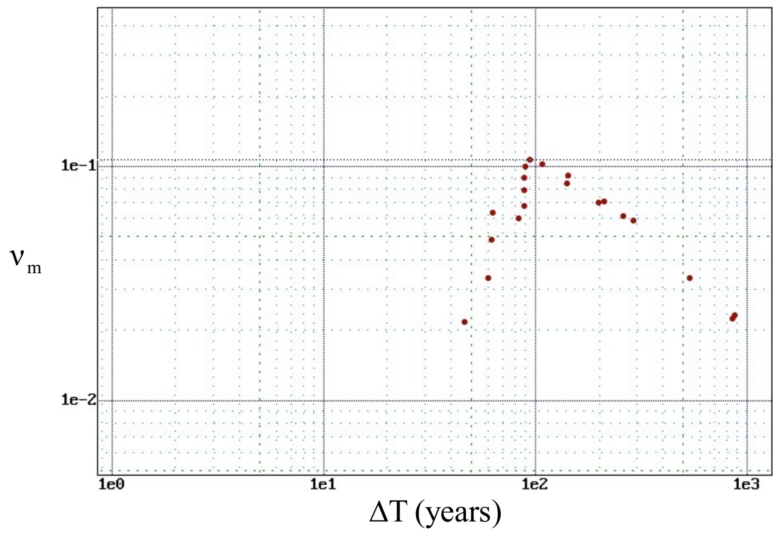

Fig.A.2: Stepp Diagram ($v_m(\Delta T)$ is the mean annual frequency of the number of earthquakes of magnitude m happened in the last ΔT years)

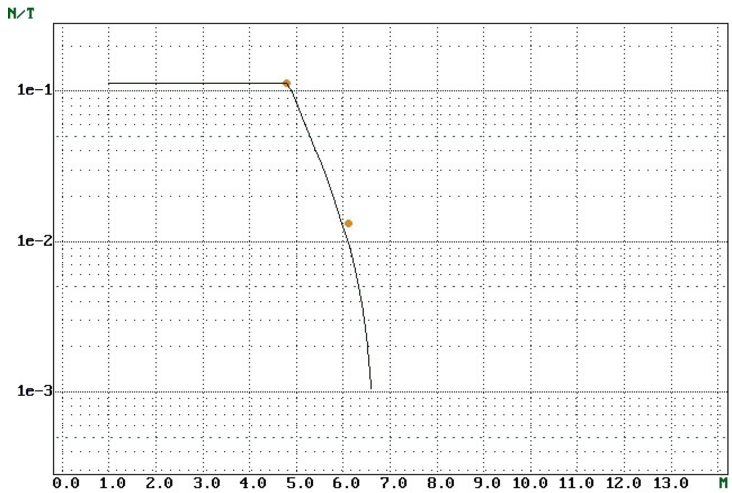

Fig.A.3: Gutenberg – Richter Bounded Law for the selected seismic zone 62 of Benevento ($v_M = N/T$ is equal to the mean annual frequency of the number of earthquakes of magnitude M)

S.Z.	49	50	51	52	53	55	56	57	58	59	60	61	62	63	64
ν_0	*0.03*	*0.35*	*0.19*	*0.14*	*0.21*	*0.17*	*0.06*	*0.11*	*0.11*	*0.07*	*0.06*	*0.25*	*0.11*	*0.34*	*0.07*
β	*1.85*	*2.46*	*1.61*	*2.26*	*1.94*	*1.35*	*1.34*	*1.06*	*1.48*	*1.19*	*2.82*	*2.529*	*1.558*	*1.774*	*2.27*
m_0	*4.4*	*4.6*	*4.6*	*4.6*	*4.5*	*4.3*	*4.7*	*4.2*	*4.6*	*4.5*	*4.3*	*4.5*	*4.8*	*4.5*	*4.5*
m_x	*5.2*	*6.7*	*7*	*6.7*	*6.2*	*6.4*	*6.2*	*5*	*7.3*	*7*	*5.1*	*6.5*	*6.7*	*7*	*7*

Table A.1: G-R Bounded Law parameters results
(S.Z.: Seismic Zone, see fig.2)

- The G.R. Bounded Law has been evaluated by regression on the Stepp diagram results.

  The results for all seismic zones selected are shown in table A.1.

II.3. Attenuation Law Parameters Evaluation

The Attenuation Law gives the IM parameter at the site of interest, as a function of magnitude m of the seismic event, of the distance r between site and epicenter, and of the site geology through the function S:

$$f(\mathrm{IM}) = a + b \cdot f_1(r) + c \cdot f_2(m) + f_3(S) \pm \sigma .$$

The form of the law, as described in [8], is the following one:

$$f(\mathrm{IM}) = a + b \cdot m + c \cdot \log_{10}\left(r^2 + h^2\right)^{1/2} + e_1 \cdot S_1 + e_2 \cdot S_2 \pm \sigma ,$$

in which S_1 and S_2 are two binary variables referring to the site classification and take the value of 1 for shallow and deep alluvium sites, respectively, and 0 otherwise, and σ is the dispersion corresponding to the uncertainty associated to the Attenuation Law.

A table giving the parameters a, b, c, h, e_1, e_2 and σ is given in [8]. Values obtained, for the case of a spectral acceleration representing the IM parameter (corresponding to T_f = 0.4 sec and ξ=0.05) and a deep alluvium site, are shown on table A.2.

A	b	c	h	e_2	σ	S_1	S_2
-2.0849	*0.445*	*-1*	*5.2*	*0.078*	*0.28*	*0*	*1*

Table A.2: Attenuation Law Parameters

II.4. Hazard Function

The seismic hazard function is evaluated by solving the following double integral:

$$P\left[S_a > s_a\right] = \sum_{i=1}^{N_s} \nu_i \cdot \int\int P\left[S_a > s_a \mid m, r\right] \cdot f_M(m) \cdot f_R(r) \cdot dm \cdot dr ,$$

where index i refers to the number of seismic zones[23];

The probability density functions of magnitude m and distance r $f_M(m)$ and $f_R(r)$ are:

$$f_R(r) = \frac{dA(R)}{A_t \cdot dR}$$

(where A_t is the total area of the seismic zone),

$$f_M(m) = \beta \cdot \frac{e^{-\beta \cdot m}}{e^{-\beta \cdot m_0} - e^{-\beta \cdot m_x}} .$$

The Attenuation Law provides the expression $P\left[S_a > s_a \mid m, r\right]$.

Parameters found on §II.2 and §II.3 have been implemented in a seismic hazard evaluation program[24]. A regression on the results yielded the k and k_0 values of §3.1.

III. Structure Modelisation

As seen in §3.2, the model of fig.5 has been used in order to perform the non-linear dynamic analyses. Section elements and vertical loads have been implemented as described in [7]. The vertical loads distribution is shown in the photo of fig.A.4 and the hysteretic stress-strain relations of concrete and steel models are shown in fig.A.5. and A.6.

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[23] Homogeneity hypothesis for each seismic zone

[24] « Mathazard », see [20].

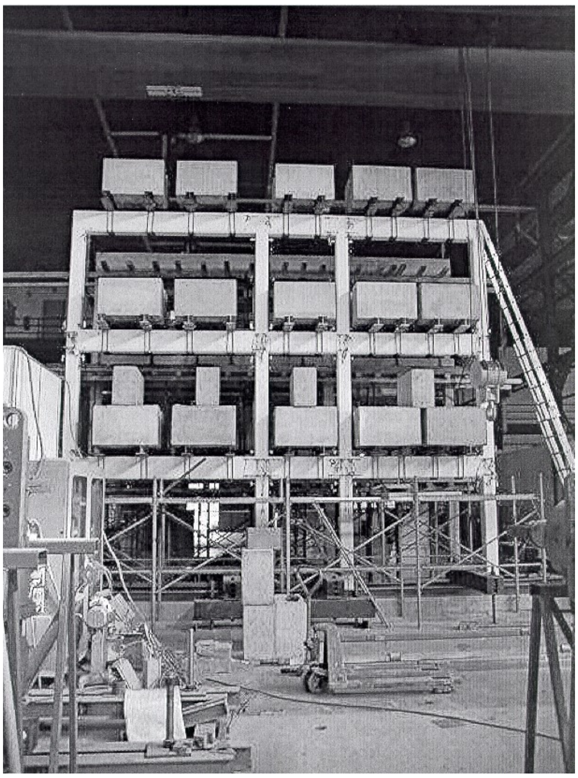

Fig. A.4: Vertical load distribution (University of Pavia Tests)

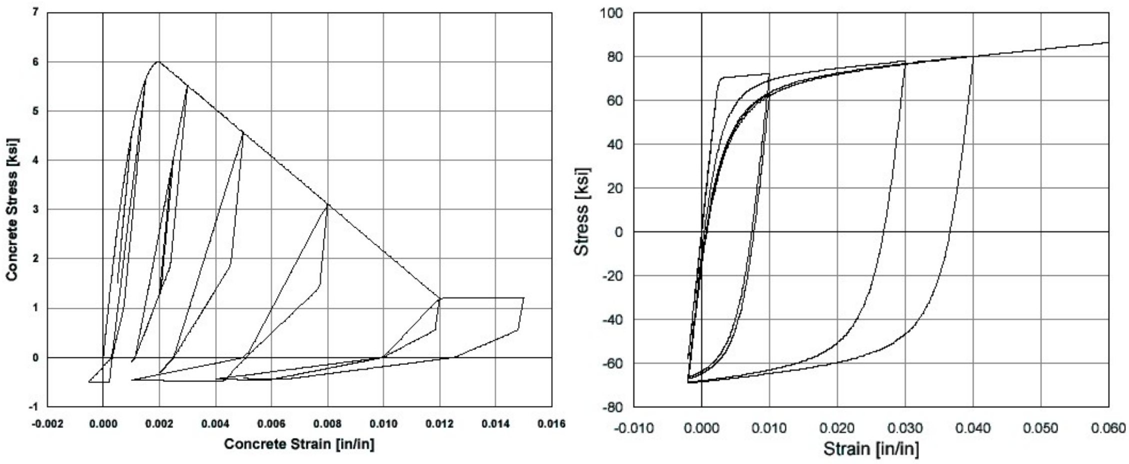

Fig. A.5 and A.6: Hysteretic Stress-Strain relation of concrete and steel models (Concrete: Kent-Park-Scott model with tensile strength and linear tension softening, steel: Giuffré-Menegotto-Pinto model with isotropic strain hardening)

Appendix References

[18] Berardi, F.: *Programma PSR.* Research Contract E.N.E.L. – Università dell'Aquila.

[19] Stepp J.C.: *Analysis of completeness of the earthquakes sample in the Puget-Sound area and its effects on statistical estimates of earthquakes hazard.* Proc. Microzonation Conf., Seattle, WA, pag. 897-909, 1972.

[20] Giannini, R.: *MATHAZARD, Un Programma per la Valutazione della Pericolosità Sismica dei Siti.* Università di Roma La Sapienza, Dicember 2000.

[21] Taylor, R.L.: *FEAP; Version 7.1 User Manual.* University of California at Berkeley, November 1998.